D–7514

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

First Semester

Mathematics

ALGEBRA – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define one to one and onto functions.

2. Define abelian group. Give an example.

3. Define normal subgroup of a group G.

4. Show that $a \in Z$ if and only if $N(a) = G$.

5. State the Pigeonhole principle.

6. Show that (3) is a prime ideal of Z.

7. Define Kernal of a homomorphism of a ring.

8. Define principal ideal ring.

9. Show that Z is an Euclidean domain where $d(a) = |a|$.

10. State the unique factorization theorem.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) For any three sets A, B, C, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Or

(b) Let $\theta : G \rightarrow H$ be onto group homomorphism with kernal K. Prove that G/K is isomorphic to H

12. (a) If n is a positive integrer and a is relatively prime to n, prove that $a^{\phi(n)} \equiv 1 \bmod n$

Or

(b) State and prove second part of sylow's theorem.

13. (a) If ϕ is a homomorphism of R into R' with kernal $I(Q)$, then prove that

(i) $I(\phi)$ is a subgroup of R under addition.

(ii) If $a \in I(\phi)$ and $\gamma \in R$ then both ar and ra are in $I(\phi)$

Or

(b) Prove that a finite integral domain is a field.

14. (a) Let R be an Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

Or

(b) Prove that the kernal of a ring homomorphism is a two sided ideal.

15. (a) Let $f(x), g(x)$ be two non zero elements of $F[x]$. Prove that $\deg(f(x).g(x)) = \deg f(x) + \deg\ g(x)$

Or

(b) Let R is an integral domain. Prove that $R[x]$ is also an integral domain.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Lagrange's theorem.

17. State and prove Cayley's theorem for abelian groups.

18. Prove that every integral domain can be imbedded in a field.

19. If v is an ideal of a ring R, then prove that R/V is a ring and is a homomorphic image of R.

20. State and prove the Eisenstein Criterian theorem.

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D–7515

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define ordered set. Give an example.

2. Define the cantor set. Give an example.

3. Give an example to show that disjoint set need not be separated.

4. When will you say a metric space is complete?

5. Define power series. Also find the radius of convergence of the power series $\sum n^3 z^n$.

6. Define bounded function.

7. State the ratio test.

8. If $\sum a_n$ converges absolutely, then prove that $\sum a_n$ converges.

9. If $f(x) = x^n$; $x \in R$, then prove that $f'(x) = n\ x^{n-1}$.

10. State the L'-Hospital's rule.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ are complex numbers, then prove that $\left|\sum_{j=1}^{n} a_j b_j\right|^2 \leq \sum_{j=1}^{n} |a_j|^2 \sum_{j=1}^{n} |b_j|^2$.

Or

(b) State and prove Weierstress theorem.

12. (a) Define e, prove that e is irrational.

Or

(b) Prove the following :

(i) If $p > 0$, then $\lim_{n \to \infty} \frac{1}{n^p} = 0$.

(ii) If $p > 0$, then $\lim_{n \to \infty} \sqrt[n]{p} = 1$.

13. (a) State and prove Merten's theorem.

Or

(b) If $\sum a_n$ is a series of complex numbers which converges absolutely, then prove that every rearrangement of $\sum a_n$ converges, and they all converges to the same sum.

14. (a) Show that monotonic functions have no discontinuities of the second kind.

Or

(b) Suppose f is a continuous mapping of a compact metric space x into a metric space y. Prove that $f(x)$ is compact.

15. (a) State and prove Taylor's theorem.

Or

(b) Examine the differentiability of the function f given by

$$f(x)=\begin{cases} x\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$$

Also verify that f is continuous at 0.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. For every real $x > 0$ and every integer $n > 0$, prove that there is one and only one real y such that $y^n = x$.

17. Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

18. Let $\sum a_n$ be a series of real numbers which converges, but not absolutely. Suppose $-\infty \leq \alpha < \beta \leq \infty$. Prove that there exists a rearrangement $\sum a_n'$ with partial sums s_n' such that $\lim_{n\to\infty} \inf s_n' = \alpha$ and $\lim_{n\to\infty} \sup s_n' = \beta$.

19. Prove that a continuous image of a connected subset of a metric space x is connected.

20. State and prove the implicit function theorem.

D–7516

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State the uniqueness theorem.

2. Define Wronskian.

3. Find the real valued solution of $y''+y=0$.

4. Prove that $P_n(-1)=(-1)^n$.

5. Compute the indicial polynomial and its roots for the differential equation $x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y=0$.

6. Define regular singular point.

7. State the second order Euler equation.

8. Compute the first four successive approximations $\varphi_0, \varphi_1, \varphi_2, \varphi_3$ for the equation $y'=y''$, $y(0)=0$.

9. Solve $y' = 3y^{2/3}$.

10. State the local existence theorem for the initial value problem $y' = f(x, y), y(x_0) = y_0$ on z.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve $y'' - 2y' - 3y = 0$, $y(0) = 0, y'(0) = 1$.

Or

(b) Find all solutions of $y'' + y = 2\sin x \sin 2x$.

12. (a) Let $L(y) = 0$ be an n^{th} order differential equation on an interval I. Show that there exist n linearly independent solution of $L(y) = 0$ on I.

Or

(b) Show that the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n (n!)^2}$.

13. (a) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\varphi_1(x) = x^2$. Find the all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

Or

(b) Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\sqrt{(1/2)}} \sin x$ and

$x^{1/2} J_{-1/2}(x) = \dfrac{\sqrt{2}}{\sqrt{(1/2)}} \cos x$.

14. (a) Let f be continuous and satisfy Lipschitz condition R. If φ and ψ are two solutions of $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I containing x_0, then prove that $\varphi(x) = \psi(x)$ for all x in I.

Or

(b) Verify whether the equation $2xy\,dx + \left(x^2 + 3y^2\right)dy = 0$ is exact or not, if exact solve it.

15. (a) Prove that between any two positive zeros of J_0 there is a zero of J_1.

Or

(b) Consider $y'_1 = 3y_1 + xy_3$, $y'_2 = y_2 + x^3 y_3$, $y'_3 = 2xy_1 - y_2 + e^x y_3$. Show that every initial value problem for this system has a unique solution which exists for all real x.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the solution φ of the initial value problem $y''' + y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.

17. With the usual notations, prove that

$$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(x/2\right)^{2m}.$$

18. Find two linear independent solution of the legendre equation $\left(1 - x^2\right)y'' - 2xy' + \alpha(\alpha + 1)y = 0$.

19. Derive Bessel functions of zero order of the first kind $J_0(x)$.

20. Let f be a real-valued continuous function on the strip $S : |x - x_0| \le a$, $|y| < \infty$, $a > 0$ and f satisfies on S a Lipschitz condition with constant $K > 0$. Prove that the successive approximations $\{\varphi_k\}$ for the problem $y' = f(x, y), y(x_0) = y_0$ exist on the entire interval $|x - x_0| \le a$ and converge there to a solution φ of $y' = f(x, y), y(x_0) = y_0$.

D–7517

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define onto function. Give an example.

2. Define dictionary order relation.

3. State the maximum principle theorem.

4. Is the subset $[a,b]$ or $\mathbb{R}$ closed? Justify your answer.

5. What are the continuous maps $f : \mathbb{R} \to \mathbb{R}_l$?

6. Define a linear continuum.

7. Define locally path connected.

8. Define first countable space.

9. Define normal space.

10. What is meant by completely regular space?

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) State and prove well-ordering property.

Or

(b) Prove that a finite product of countable set is countable.

12. (a) Let Y be a subspace of X. Prove that a set A is closed in Y if and only if equals the intersection of a closed set of X with Y.

Or

(b) State and prove the sequence lemma.

13. (a) State and prove the intermediate value theorem.

Or

(b) Let A be a connected subspace of X, If $A \subset B \subset \overline{B}$, then prove that B is also connected.

14. (a) Show that every closed subspace of a compact space is compact.

Or

(b) Prove that a subspace of a completely regular space is completely regular.

15. (a) Prove that every metrizable space with a countable dense subset has a countable basis.

Or

(b) Prove that a product of regular space is regular.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Define countable and uncountable set. Prove that the set $\rho\ (\mathbb{Z}_+)$ of all subsets of $\mathbb{Z}_+$ is uncountable.

17. Prove that the collection

$S = \{\pi_1^{-1}(\mho)/\mho \text{ is open in } X\} \cup \{\pi_2^{-1}(V)/V \text{ is open in } Y\}$ is a subbasis for the product topology on $X \times Y$.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L.

19. Let X be a simply ordered set having least upper bound property. Prove that in the order topology, each closed interval in X is compact.

20. State and prove Urysohn lemma.

D–7518

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

Second Semester

Mathematics

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. When will you say a vector space is a homomorphism?

2. Define the linear span.

3. Define the fixed field of a group automorphism.

4. What is meant by the Galois group of $f(x)$?

5. Define an invertible element.

6. Define a characteristic vector of T .

7. Define hermition and skew hermition.

8. Define companion matrix of $f(x)$.

9. Write the rational canonical form of the matrix of T in $A_F(V)$.

10. Define the range of T.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If V is the internal direct sum of $v_1, v_2, \ldots\ldots, v_n$, then prove that V is isomorphic to the external direct sum of $v_1, v_2, \ldots\ldots, v_n$.

Or

(b) Prove that $A(W)$ is a subspace of V.

12. (a) State and prove the Bessel's inequality.

Or

(b) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

13. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) If K is a finite extension of F, prove that $G(K,F)$ is a finite group and its order $o(G(K,F))$ satisfies $o(G(K,F)) \leq [K:F]$.

14. (a) If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

Or

(b) If $(vT, vT) = (v, v)$ for all $v \in V$ then prove that T is unitary.

15. (a) If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then prove that $T = 0$.

Or

(b) Let $T \in A(V)$ be Hermition prove that all its characteristic roots are real.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If V is finite dimensional and if W is a subspace of V, then prove that W is finite dimensional, $\dim W \leq \dim V$ and $\dim \left(V/W\right) = \dim V - \dim W$.

17. Derive the Gram - Schmidt orthogonolisation process.

18. Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

19. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

20. For $A, B \in F_n$, Prove that $\det(AB) = (\det A)(\det B)$.

D–7519

Sub. Code
31122

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Riemann-stielties of a bounded real function f on $[a,b]$.

2. Define refinement of a partition.

3. When will you say a function is uniformly bounded?

4. Define rectifiable curve.

5. On what intervals does the series $f(x)=\sum_{n=1}^{\infty}\frac{1}{1+n^2x}$ fail to converge uniformly?

6. Define fourier series.

7. show that the measurable set E is periodic.

8. When will you say a function is Borel measurable?

9. Define Simple function.

10. Let f be a non negative measurable function. Show that $\int f = 0$ implies $f = 0$ almost every where.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b)

11. (a) Prove that $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\in > 0$, there exists a partition P such that $V(P,f,\alpha) - L(P,f,\alpha) < C$.

Or

(b) State and prove schwarz inequality.

12. (a) If K is a compact metric space, if $f_n \in \mathcal{C}(k)$ for $n = 1,2,..,$ and if $\{f_n\}$ converge uniformly on k, then prove that $\{f_n\}$ is equi continuous on k.

Or

(b) Let $a_o, a_1, .. a_n$ be complex numbers, $n \geq 1, a_n \neq 0$ such that $p(z) = \sum_{k=0}^{n} a_k z^k$, prove that $p(z) = 0$ for some complex number z.

13. (a) Prove that, if $x > 0$ and $y > 0$ then $\int_0^1 f^{x-1}(1-t)^{y-1} dt = \frac{\sqrt{x}\sqrt{y}}{\sqrt{x+y}}$

Or

(b) Define Borel set. Prove that every Borel set is measurable.

14. (a) State and prove Lusin's theorem.

Or

(b) If A and B are disjoint measurable sets of finite measure, then prove that $\int_{A \cup B} f = \int_A f + \int_B f$.

15. (a) State and prove Fatou's theorem

Or

(b) State and prove bounded convergence theorem.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b / \gamma'(t) / dt$

17. State and prove the stone-weierstrass theorem

18. Prove

(a) For every $A \in \mathcal{G}$, $\mu * (A) = \mu(A)$

(b) If $E = \bigcup_{n=1}^{\infty} E_n$, then $\mu * (E) \leq \sum_{n=1}^{\infty} \mu * (E_n)$.

19. Prove that the outer measure of an interval is its length.

20. State and prove Lebesgue's dominated convergence theorem.

D–7520

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

Second Semester

Mathematics

TOPOLOGY – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a G_5 – set in a space. Give an example.
2. Define one point compactification.
3. Define countably locally finite.
4. Define locally metrizable space.
5. Give an example of a Cauchy sequence in Q that is not convergent in it.
6. Define the point open topology.
7. Define the evaluation map.
8. Define a Baire space. Give an example.
9. Define a equicontinuous space.
10. What is meant by topological dimension?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a product of a completely regular space is completely regular.

Or

(b) Let $A \subset X$; let $f : A \to Z$ be a continuous map of A into the Hausdorff space Z. Prove that there is atmost one extension of f to a continuous function $g : \overline{A} \to Z$.

12. (a) Let X be a topological space. Prove that the set $\mathcal{B}(X, R)$ of all bounded functions $f : x \to R$ is complete under the sub metric J.

Or

(b) Show that the metric (x, d) is complete if and only if for any nested sequence $A_1 \supset A_2 \supset \ldots$ of non empty closed sets of X such that diameter $A_n \to 0, \bigcap A_n \neq \varphi$ $n \in z_+$.

13. (a) Prove that every para-compact space X is normal.

Or

(b) If X is locally compact, or if X satisfies the first countability axiom, then prove that X is compactly generated.

14. (a) Prove that every metrizable space is para compact.

Or

(b) Show that every open subset of a Baire space is a Baire space.

15. (a) Define $F_\sigma -$ set. Also prove that W is an $F_\sigma -$ set in X if and only if $X - W$ is a $G_\delta -$ set.

Or

(b) Show that any compact subset C of R^2 has topological dimension at most 2.

PART C— (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tychonoff theorem.

17. State and prove the Peano space-filling curve.

18. Prove that a metric space (x, d) is compact if and only if it is complete and totally bounded.

19. State and prove Ascoli's theorem.

20. Let $X = Y \cup Z$, where Y and Z are closed sets in X having finite topological dimension. Prove that $\dim X = \max\{\dim Y, \dim z\}$.

D–7521

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Show that $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ is integrable.

2. Define orthogonal trajectories of a system of curves on a surface.

3. Find the complete integral of $pq = 1$.

4. Eliminate the arbitrary function f from the equation $z = f(x^2 + y^2)$.

5. Find the partial differential equation by the elimination of a and b from $z = (x + a)(y + b)$

6. Write down the fundamental idea of Jacob's method.

7. When we say that the equation $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ is elliptic.

8. Solve $(D^2 - D')z = 0$.

9. Write down the interior Dirichlet boundary value problem for Laplace's equation.

10. Write down the exterior Neumann boundary value problem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1, x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

Or

(b) Show that $y\,dx + x\,dy + 2z\,dz = 0$ is integrable and hence solve.

12. (a) Eliminate the arbitrary function f from $z = xy + f(x^2 + y^2)$.

Or

(b) Find the general integral of the partial differential equation $z(xp - yq) = y^2 - x^2$.

13. (a) Find the general solution of the partial differential equation $(2xy-1)\,p + (z-2x^2)q = 2(x-yz)$ and also the particular integral which passes through the line $x = 1, y = 0$.

Or

(b) Using Jacobi's method, solve $xp^2 + yq^2 = z$.

14. (a) Solve $v + s - 2t = e^{x+y}$.

Or

(b) Solve $(D'^2 + 2kD' - C^2D^2)y = 0$.

15. (a) Derive D'Alembert's solution of the one-dimensional wave equation.

Or

(b) Is it possible to reduce the Neumann problem to Dirichlet problem? Justify.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Verify that the equation $z(z+y^2)dx + z(z+x^2)dy - xy(x+y)dz = 0$ is integrable and find its primitive.

17. Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible, and solve them.

18. Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} = x^2 \dfrac{\partial^2 z}{\partial y^2}$ to canonical form.

19. By separating the variables, solve the one dimensional diffusion equation. $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

20. Heat flows in a semi-infinite rectangular plate, the end $x = 0$ being kept at temperature θ_0 and the long edge $y = 0$ and $y = a$ at zero temperature. Prove that the temperature at a point (x, y) is

$$\frac{4\theta_0}{\pi}\sum_{n=0}^{\infty}\frac{1}{2m+1}\sin\left(\frac{(2m+1)\pi y}{a}\right)e^{-(2m+1)\frac{\pi x}{a}}.$$

D–7522

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2022.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define Torsion of curve.

2. What is meant by rectifying plane?

3. Define point of inflexion.

4. What are the intrinsic equations of the curve?

5. Define class of a surface.

6. Define the pitch of the helicoid.

7. Define geodesic parallels.

8. State the necessary and sufficient condition that the curve $v = c$ be a geodesic.

9. State the canonical geodesic equation.

10. Define a developable surface.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If the radius of spherical curvature is constant, then prove that the curve either lies on a sphere or has constant curvature.

Or

(b) Prove that the involute of a circular helix are plane curves.

12. (a) If a curve lies on a sphere, show that σ and ρ are connected by $\frac{\rho}{\sigma} + \frac{d}{ds}(\sigma\rho') = 0$.

Or

(b) Prove that a characteristic property of helix is that the ratio of the curvature to the torsion is constant.

13. (a) Prove that the metric is invariant under a parameter transformation.

Or

(b) Find the co-efficient of the direction which makes an angle $\pi / 2$ with direction where co-efficients are (l, m).

14. (a) Prove that every helix on a cylinder is geodesic.

Or

(b) Prove that on a general surface, a necessary and sufficient condition that the curve $v = c$ be a Geodesic is $EE_2 + FE_1 - 2EF_1 = 0$.

15. (a) Write a brief note on second fundamental form of a surface.

Or

(b) State and prove Meusnier's theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadratic surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$.

17. State and prove uniqueness theorem.

18. (a) Find the differential equations of the orthogonal trajectories of a given family of curves on a given surface.

(b) If θ is the angle of the points uv between the two direction du, dv given by $pdu^2 + 2Qdudv + Rdv^2 = 0$, then prove that $\tan\theta = \frac{2H(Q^2 - PR)^{1/2}}{ER - 2FQ + GP}$.

19. Derive the geodesic differential equations.

20. State and prove the Monge's theorem.

D–7523

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define path, spanning tree.

2. Define cut capacity and event.

3. Define PERT and Float.

4. Define basic solution.

5. Define the optimality condition of the revised simplex method.

6. Define pay-off matrix.

7. Define strong maxima.

8. Define sensitivity coefficients.

9. Define absolute maximum.

10. Define Separable.

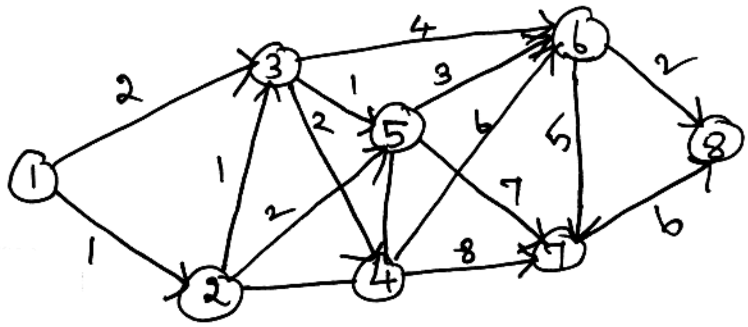
PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write down the minimal spanning tree algorithm.

Or

(b) Use Dijkstra's algorithm to determine the optimal solution of the following problem.



12. (a) Determine all basic feasible solution of the following system of equations $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$.

Or

(b) Determine the strategies that define the saddle point and the value of the game.

	B_1	B_2	B_3	B_4
A_1	5	–4	–5	6
A_2	–3	–4	–8	–2
A_3	6	8	–8	–9
A_4	7	3	–9	6

13. (a) Determine the extreme point of the following function

$$f(X) = 2x_1^2 + x_2^2 + x_3^2 + +6(x_1 + x_2 + x_3) + 2x_1x_2x_3 .$$

Or

(b) Solve by Jacobian method

Minimize $f(X) = 5x_1^2 + x_2^2 + 2x_1x_2$

Subject to

$g(X) = x_1x_2 - 10 = 0$

$x_1, x_2 \geq 0$

14. (a) Explain the constrained algorithm.

Or

(b) Write down the sufficient condition of the KKT conditions.

15. (a) Construct the project network

Activity	Predecessor (s)	Duration	Activity	Predecessor (s)	Duration
A	-	3	I	H	3
B	A	14	J	H	2
C	A	1	K	I, J	2
D	C	3	L	K	2
E	C	1	M	L	4
F	C	2	N	L	1
G	D, E, F	1	O	B, M, N	3
H	G	1			

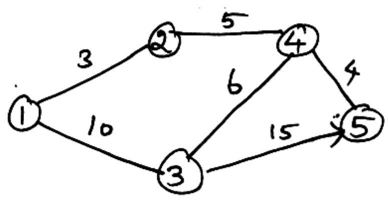
Or

(b) Explain the critical path method.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. For the following network find the shortest route between every two nodes.



17. Solve the following LPP by revised simplex method
Minimize $Z = 2x_1 + x_2$
Subject to
$3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$
$x_1 + 2x_2 \leq 3$
$x_1, x_2 \geq 0$

18. Solve the following game by Linear programming technique

		Player Q		
		Q_1	Q_2	Q_3
Player P	P_1	9	1	4
	P_2	0	6	3
	P_5	5	2	8

19. Solve $f(x) = (3x-2)^2\ (2x-3)^2$ by Newton Raphson method.

20. Solve the following LPP by Lagrangean method
Minimize $f(X) = x_1^2 + x_2^2 + x_3^2$
Subject to
$g_1(X) = x_1 + x_2 + 3x_3 - 2 = 0$
$g_2(X) = 5x_1 + 2x_2 + x_3 - 5 = 0$

D–7524

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2022.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. If $(a, b) = 1$ and if $c \mid a$ and $d \mid b$, then show that $(c, d) = 1$.

2. Find the g.c.d. of 826, 1890.

3. Define Mangoldt function $\wedge$ and write down the values of $\wedge(n)$, for $1 \leq n \leq 5$.

4. Define Multiplicative functions.

5. If f and g are arithmetical functions then prove that $(f * g)' = (f' * g) + (f * g')$.

6. Define the big *oh* notation and asymptotic equality of functions.

7. State the Little Fermat's theorem.

8. Solve the congruence $25x \equiv 15 \pmod{120}$.

9. Define Legendre's symbol. Give an example.

10. Determine whether –104 is a quadratic residue or non-residue of the prime.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

Or

(b) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

12. (a) For any two arithmetical functions f and g, let $h = f * g$. Prove that, for every prime p, $h_p(x) = f_p(x) g_p(x)$.

Or

(b) Prove that for every $n \geq 1$, $\sum_{d|n} \lambda(d) = \begin{cases} 1 & \text{if } n \text{ is a square} \\ 0 & \text{Otherwise} \end{cases}$ Also prove that $\lambda^{-1}(n) = |\mu(n)|$ for all n.

13. (a) If f is multiplicative then prove that $\sum_{d|n} \mu(d) f(d) = \prod_{p|n} (1 - f(p))$.

Or

(b) State and prove Generalized inversion formula.

14. (a) If a and b are positive real numbers such that $ab = x$ then prove that

$$\sum_{\substack{q,\, d \\ qd \le x}} f(d)\, g(d) = \sum_{n \le a} f(n)\, G\left(\frac{x}{n}\right) + \sum_{n \le b} g(n)\, F\left(\frac{x}{n}\right) - F(a) G(b).$$

Or

(b) State and prove Euler-Fermat theorem.

15. (a) State and prove Chinese remainder theorem.

Or

(b) If P and Q are positive odd integers with $(P, Q) = 1$ then prove that $\left(\frac{P}{Q}\right)\left(\frac{Q}{P}\right) = \frac{(-1)^{(p-1)(Q-1)}}{4}$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. (a) State and prove Euclidean algorithm.

(b) Prove that if $n \ge 1$, $\sum_{d|n} \varphi(d) = n$.

17. State and prove product form of the Mobius inversion formula.

18. Let f be multiplicative. Prove that f is completely multiplicative if and only if, $f^{-1}(n) = \mu(n) f(n)$, for all $n \ge 1$.

19. (a) Prove that congruence is an equivalence relation.

(b) Assume $(a, m) = 1$. Then prove that the linear congruence $ax \equiv b \pmod{m}$ has exactly one solution.

20. State and prove Gauss Lemma.

D–7525

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define null persistent state.

2. Define Markov chain.

3. Define residual time.

4. Define diffusion processes.

5. Write the backward diffusion equation.

6. Define Ornstein-Vhlenbeck process.

7. Define service time.

8. State the total number of progeny.

9. Define Idle period.

10. Write the Erlang's loss formula.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2, with transition matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution $\Pr(X_0 = i) = \frac{1}{3}, i = 0, 1, 2$

Find

(i) $\Pr(X_2 = 2, X_1 = 1 / X_0 = 2)$

(ii) $\Pr(X_2 = 2, X_1 = 1, X_0 = 2)$

Or

(b) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that $\Pr\{N(s) = k / N(t) = n\} = \binom{n}{k}\left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$

12. (a) If $X(t)$, with $X(0)$ and $\mu = 0$, is a Wiener process, show that $Y(t) = \sigma X\left(\frac{t}{\sigma^2}\right)$ is a Wiener process. Find its covariance function.

Or

(b) Let $X(t)$ be the displacement process corresponding to the velocity $\mu(t)$ show that in equilibrium position $E[X(t) - X(0)] = 0$ and $Var[X(t) - X(0)] = \sigma^2(\beta t + e^{-\beta t} - 1)/\beta^3$.

13. (a) For a G.W. process with $m=1$, $\sigma^2 < \infty$, prove that $\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{1-P_n(s)} - \frac{1}{1-s}\right\} \to \frac{\sigma^2}{2}$ uniformly in $0 \le s < 1$.

Or

(b) Find the variance of X_n in a G.W process from the relation $P_n = P(P_{n-1}(s))$

14. (a) Derive Little's formula.

Or

(b) For $r, n = 0, 1, 2, \ldots\ldots;$ prove that $E\{E_{n+r} / X_n\} = X_n m^r$.

15. (a) Show that the expected number of busy servers in an $M/M/C$ queue in steady state is $CP = \lambda/\mu$.

Or

(b) Derive Pollaczek-Khinchine formula.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Prove that the p.g.f. of a non-homogeneous process $\{N(t), t \ge 0\}$ is $Q(s,t) = \exp\{m(t)(s-1)\}$, where $m(t) = \int_0^1 \lambda(x)\,dx$ is the expectation of $N(t)$.

17. If the intervals between successive occurrences of an event E are independently distributed with a common exponential distribution with mean $\frac{1}{\lambda}$, then prove that the events E form a Poisson process with mean λt.

18. Discuss briefly and derive Ornstein-Vhlenbeck process.

19. State and prove Yaglom's theorem.

20. A mechanic looks after 8 automatic machines, a machine breaks down, independently of others, in accordance with a Poisson process, the average length of time for which a machine remains in working order being 12 hours. The duration of time required for repair of a machine has an exponential distribution with mean 1 hour. Find

    (a) The probability that 3 or more machines will remain out of order at the same time.

    (b) For what fraction of time, on the average, the mechanic will be idle and

    (c) The average duration of time for which a machine is not in working order.

D–7526

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2022.

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define degree of a vertex in a graph.

2. Define walk and path of a graph.

3. Define edge connectivity. Give an example.

4. Define the edge chromatic number.

5. Prove that in a tree any two vertices are connected by a unique path.

6. Find the number of different perfect matchings in k_n.

7. If G is a simple planar graph with $\gamma \geq 3$, then prove that $\varepsilon \leq 3\gamma - 6$.

8. Using Euler's formula, prove that, if G is a simple planar graph, then $\delta \leq 5$.

9. Define isomorphic directed graphs.

10. Define directed path and directed cycles.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In any graph G, prove that the number of vertices of odd degree is even.

Or

(b) Show that $k_p - v = k_{p-1}$ for any vertex v of k_p.

12. (a) If $\delta \geq k$ then prove that G has a path of length K.

Or

(b) Let G be a K-regular bipartite.

13. (a) Prove that $\gamma(m,n) = \gamma(n,m)$.

Or

(b) If G is bipartite then prove that $\chi' = \Delta$.

14. (a) Show that K_5 is not planar.

Or

(b) When will you say a graph is self dual? Also prove that if G is self dual, then $\varepsilon = 2V = 2$.

15. (a) Prove that a graph G is strongly orientable if and only if G has no cut edge.

Or

(b) Prove that every tournament has a spanning path.

PART C— (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that $\tau(k_n) = n^{n-2}$.

17. State and prove Vizing's theorem.

18. State and prove Euler's formula for a connected plane graph.

19. State and prove Brook's theorem.

20. Prove that a weak digraph D is Eulerian if and only if every point of D has equal indegree and outdegree.

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D–7527

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define normed space. Give an example.

2. Define orthogonal transformation.

3. Define the dimension of a Hilbert space.

4. Let X be a linear space over c and u a real-linear functional on X. Define $f(x) = ux - iu(ix)$, $x \in X$. Prove that f is a complex linear functional on X.

5. Write a note on Banach limit.

6. Define annihilator.

7. Define unitary and normal operators.

8. Show that l^2 is reflexive.

9. State the open mapping theorem,

10. Let $x \in X$, X a linear space. Prove that $< x, y >= 0$ for all $y \in X$ if and only if $x = 0$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (X,d) and (Y,d') are metric spaces where $f: X \to Y$, then prove that f is continuous at x if and only if for every sequence $\{x_n\}$ converging to x, $f(x_n) \to f(x)$.

Or

(b) Prove that norm is a continuous function.

12. (a) Let X be a normed linear space and let k be a convex subset of X. Prove that $\overline{K}$ is convex.

Or

(b) Let X and Y be normed linear spaces. Let $T: X \to Y$ be a linear transformation. Prove that T is continuous if and only if T is continuous at the origin.

13. (a) Prove that a normed space with Schauder basis is separable.

Or

(b) Prove that every inner product space is a normed linear space.

14. (a) Prove that any two orthonormal bases in a Hilbert space have the same cardinality.

Or

(b) Let $\widetilde{X}$ be the dual space of the normed linear space X. Prove that if $\widetilde{X}$ is separable, so is X.

15. (a) Suppose $A : X \to Y$. If A is completely continuous then prove that the range of A, $R(A)$ is separable.

Or

(b) Show that a completely continuous transformation maps a weakly convergent sequence into a strongly convergent one.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Prove that a subset A of a metric space (X, d) is sequentially compact if and only if it is compact.

17. Let X be an inner product space, A an orthonormal set of vectors in X, and y an arbitrary vector in X. Then prove that

(a) for all $x_1, x_2 \ldots, x_n \in A$, $\sum_{i=1}^{n} \left| (y, x_i) \right|^2 \leq \| y \|^2$

(b) the set $E = \{x \in A / (y, x) \neq 0\}$ is countable

(c) if $z \in X$, then $\sum_{x \in A} \left| (y, x) \overline{(z, x)} \right| \leq \| y \| \cdot \| z \|$.

18. State and prove Riesz representation theorem for a linear functionals on a Hilbert space.

19. State and prove uniform boundedness theorem.

20. State and prove closed graph theorem.

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D–7528

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2022.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a multiple root of multiplicity m.

2. What is meant by strum sequence?

3. Define eigen vector.

4. What is meant by truncation error?

5. State the Hermite interpolating polynomial.

6. Write the Lagrange bivariate interpolating polynomial.

7. Define Gauss elimination method.

8. What is the order of the error in Simpson's formula?

9. What are the two types of errors in numerical differentiation?

10. Write the advantages of R-K model.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve $x^3 - 5x + 1 = 0$ by the Newton-Raphson method.

Or

(b) Use synthetic division and perform two iterations of the Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$.

12. (a) Solve the following equations using the Gauss elimination method.

$$10x - y + 2z = 4$$
$$x + 10y - z = 3$$
$$2x + 3y + 20z = 7.$$

Or

(b) Find the largest eigen value in modulus and the corresponding eigen vectors of the matrix.

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

13. (a) Given that

x :	0	1	2	3
$f(x)$:	1	2	33	244

Fit quadratic splines with $M(0) = f''(0)$. Hence, find an estimate of $f(2.5)$.

Or

(b) Obtain the least squares straight line fit to the following data :

x :	0.2	0.4	0.6	0.8	1
$f(x)$:	0.447	0.632	0.775	0.894	1

14. (a) A differentiation rule of the form

$hf'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$ where $x_j = x_0 + jh,\ j = 0,1,2,3,4$ is given. Determine the values of $\alpha_0,\ \alpha_1,\ \alpha_2,\ \alpha_3$ so that the rule is exact for a polynomial of degree 4 and find the error term.

Or

(b) Determine $\alpha, \beta, \gamma, \delta$ so that the relation $y'((a+b)/2) = \alpha y(a) + \beta y(b) + \gamma y''(a) + \delta y''(b)$ is exact for polynomials of as high degree as possible. Give an asymptotically valid expression for the truncation error as $|b-a| \to 0$.

15. (a) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using trapezoidal rule and Simpson's rule.

Or

(b) Evaluate $\int_0^\infty \frac{e^{-x}}{1+x^2}\,dx$, using the Gauss-Laguerre two point formula.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the interval in which the smallest positive root of the equation $x^3 - x - 4 = 0$ lies. Determine the roots current to two decimal places using the bisection method.

17. Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $p_3(x) = x^3 + x^2 - x + 2 = 0$. Use the initial approximations $p_0 = -0.9, q_0 = 0.9$.

18. Construct the Hermite interpolation polynomial that fits the data

x	$f(x)$	$f'(x)$
0	4	–5
1	–6	–14
2	–22	–17

Interpolate $f(x)$ at $x = 0.5$ and $x = 1.5$.

19. Solve the initial value problem $u' = -2tu^2, u(0) = 1$ using the midpoint method, with $h = 0.2$, over the interval $[0, 1]$. Use Taylor series method of second order to compute $u\ (0.2)$.

20. Using fourth order Runge-Kutta method find $u(0.2)$ from $u' = u - t,\ u(0) = 2$ taking $h = 0.1$.

D–7529

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2022.

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Find $P(A/B)$ and $P(B/A)$ if $P(A) = 0.4, P(B) = 0.5$ and $P(A \cap B) = 0.3$.

2. Define the conditional expectation.

3. Define moment generating function.

4. Define an exponential distribution.

5. If the moment generating function of a random variable X is $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^5$, find $P(x = 2 \text{ or } 3)$.

6. Write down the mean and variance of the beta distribution.

7. Let X be a $N(2.25)$ Find $P_r(0 < x < 10)$.

8. Let the independent random variables x_1 and x_2 have the same p.d.f. $f(x) = \begin{cases} \frac{x}{6}, & x = 1, 2, 3 \\ 0 & elsewhere \end{cases}$, find $P_r(x_1 = 2, x_1 = 3)$.

9. Let y be b $\left(72, \frac{1}{3}\right)$. Find $P_r(22 \leq y \leq 28)$.

10. Let z_n be $x^2(n)$. Find the mean and variance of z_n.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Baye's theorem.

Or

(b) A bowl contains seven blue chips and three red chips. Two chips are to be drawn successively at random and without replacement. Compute the probability that the first draw results in a red chip (A) and the second draw results in a blue chip (B).

12. (a) Prove that

(i) $E[E(x_2 / x_1)] = E(x_2)$

(ii) $Var\,[E(x_2 / x_1)] \leq \text{var}\,(x_2)$

Or

(b) Let the joint p.d.f. of x_1 and x_2 be

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, 0 < x_1 < 1; & 0 < x_2 < 1 \\ 0, & elsewhere \end{cases}$$

Show that the random variables x_1 and x_2 are dependent.

13. (a) Derive the mean and variance of a gamma distribution.

Or

(b) Let x equal a tarsus length for a male grackle. Assume that the distribution of x is $N(\mu, 4.84)$. Find the sample size n that is needed so that we are 95% confident that the maximum error of the estimate of μ is 0.4.

14. (a) Let $\overline{X}$ be the mean of a random sample of size 12 from the uniform distribution on the interval (0,1). Find $P(1/2 \leq \overline{X} < 2/3)$.

Or

(b) Let x have the uniform distribution over the interval $(-\pi/2, \pi/2)$. Show that $y = \tan x$ has a cauchy distribution.

15. (a) Let the p.d.f. Y_n be $f_n(y) = \begin{cases} 1, & y = n \\ 0, & elsewhere \end{cases}$

show that Y_n does not have a limiting distribution.

Or

(b) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) State and prove Chebyshev's inequality.

(b) Prove that $E[(x - \mu_1)(y - \mu_2)] = E(xy) - \mu_1 \mu_2$

17. Let x_1 and x_2 have the joint p.d.f.

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x < 1, 0 < x_2 < 1 \\ 0, & elsewhere \end{cases}$$

Find

(a) $P(x_1 \leq 1/2)$

(b) $P_r(x_1 + x_2 \leq 1)$

18. Let S^2 be the variance of random sample of size 6 from the normal distribution $N(\mu, 12)$. Find $P(2.30 < S^2 < 22.2)$

19. Find the p.d.f. of the beta distribution in which α and β are parameters.

20. State and prove central limit theorem.